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ADVANCING SCIENCE

Digitised Mathematics for the Working Mathematician

Klaus Hulek, DMV-ÖMG-Jahrestagung, Sep 27th, 2021

The basic question

Thanks go to Olaf Teschke for discussions and support

Basic question:

- What are the needs of a working mathematician/the mathematical community in the age of digitization?
- Which (digital) infrastructures do we need?

Background

Different viewpoints

The question asked above can and will have different answers depending on its context.

Personal background

- Working research mathematician (since 1977) and professor
- Editor of a journal and various proceedings
- Evaluator (appointment committees, funding agencies, institutional evaluations)
- Vice president for research of Leibniz Universität Hannover (for 9 years)
- Editor-in-Chief of zbMATH Open (provider of digital information)

Main thesis

Claim

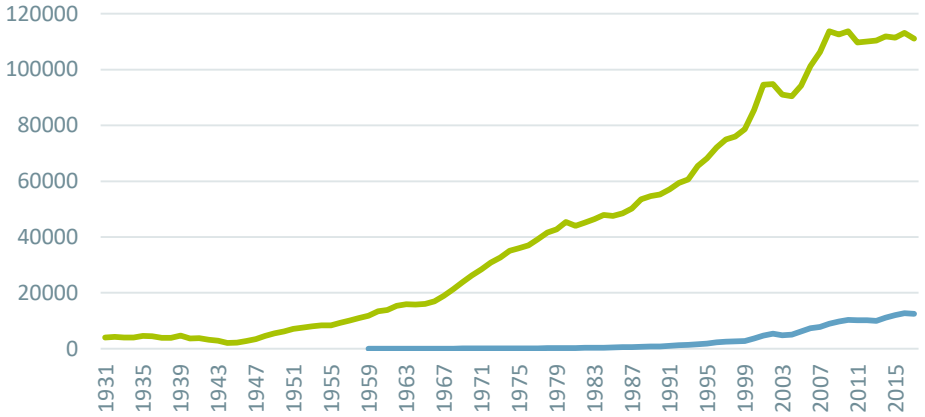
The basic features

- Mathematical research and teaching
- Dissemination of mathematical results
- Quality control of mathematical publications
- Evaluations of peers and institutions

have not changed significantly (altough the importance of evaluations has increased). What has changed is a dramatic increase in the **amount**, **complexity,** and **diversity** of the available information.

Growing amount, complexity, and diversity of information

zbMATH indexed publications and software



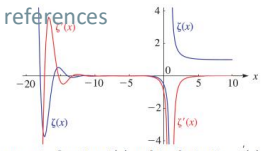


Figure 25.3.1: Riemann zeta function $\zeta(x)$ and its derivative $\zeta'(x)$, $-20 \le x \le 10$.

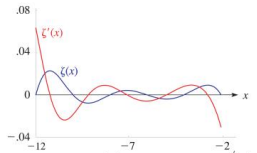


Figure 25.3.2: Riemann zeta function $\zeta(x)$ and its derivative $\zeta'(x)$, $-12 \le x \le -2$.

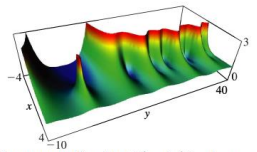
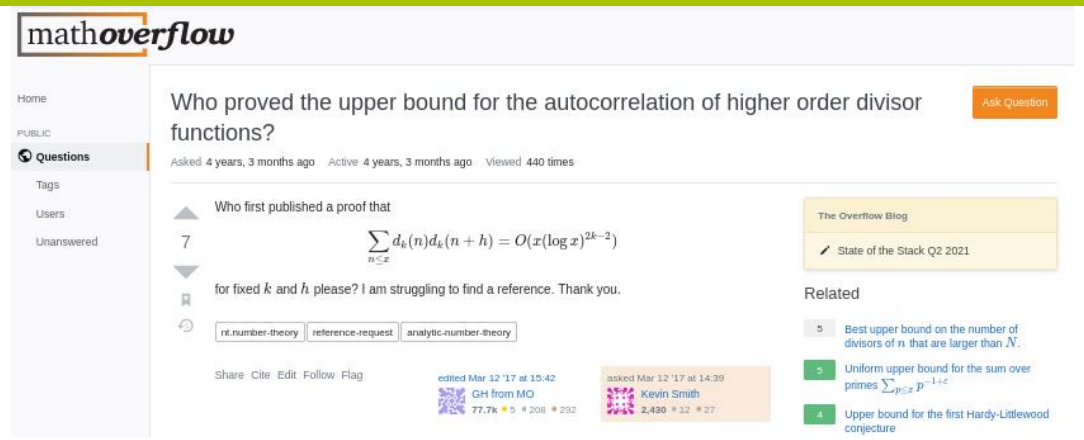


Figure 25.3.3: Modulus of the Riemann zeta function $|\zeta(x+iy)|$, $-4 \le x \le 4$, $-10 \le y \le 40$. Help



A300194 Coefficients of non-constant terms of a Calabi-Yau modular form attached to 4-dimensional Dwork family.

1, 4131, 51734044, 918902851011, 19562918469120126, 465569724397794578388, 11949937773734987494551484O, 32396875735511780391532989869l,
9154540571018908853569832253702901, 267225273114956122025822445917802760506 (list; graph; refs; listen; history; text; internal format)

OFFSET 1,2

COMMENTS The 8-tuple (1/36 + 20*A300194, -1 + 216*A300195, -1/36 + 14*A300196, -1/6 + 24*A300197, -1/72 + 2*A300198, -1/46656 * A300199, 1/36 - 2*A300200, -1/7776 + 7/18 * A300201) gives a solution of the modular vector field R = Sum_{i=1..8} R_i d/dt_i on the enhanced moduli space arising from 4-dimensional Dwork family, where d/dt_i's give the standard basis of the tangent space in the chart (t_1,t_2,...,t_8) and

```
R_1 = -t_1*t_2+t_3;
R_2 = (-t_1^6*t_2^2+1/36*t_3^2*t_4*t_8+t_2^2*t_6)/(t_1^6-t_6);
R_3 = (-3*t_1^6*t_2*t_3+1/36*t_3^2*t_5*t_8+3*t_2*t_3*t_6)/(t_1^6-t_6);
R_4 = (-t_1^6*t_2*t_4-1/36*t_3^2*t_7*t_8+t_2*t_4*t_6)/(t_1^6-t_6);
R_5 = (-2*t_1^6*t_3*t_4-4*t_1^6*t_2*t_5+5*t_1^4*t_3*t_8+1/36*t_3*t_5^2*t_8+ 2*t_3*t_4*t_6+4*t_2*t_5*t_6)/(2*
  (t_1^6-t_6));
R_6 = -6*t_2*t_6;
R_7 = -18*t_1^2+1/2*t_4^2;
R_8 = (-3*t_1^6*t_2*t_8+3*t_1^5*t_3*t_8+3*t_2*t_6*t_8)/(t_1^6-t_6);
For more details see the Movasati & Nikdelan link Section 8.3.
```

LINKS Table of n, a(n) for n=1..10.
H. Movasati, Y. Nikdelan, Gauss-Manin Connection in Disguise: Dwork Family, arXiv:1603.09411 [math.AG], 2016-2017. See Table 2, (1/20)*t_1.
H. Movasati, Foliation.lib.

PROG (SINGULAR)

zbMATH Open

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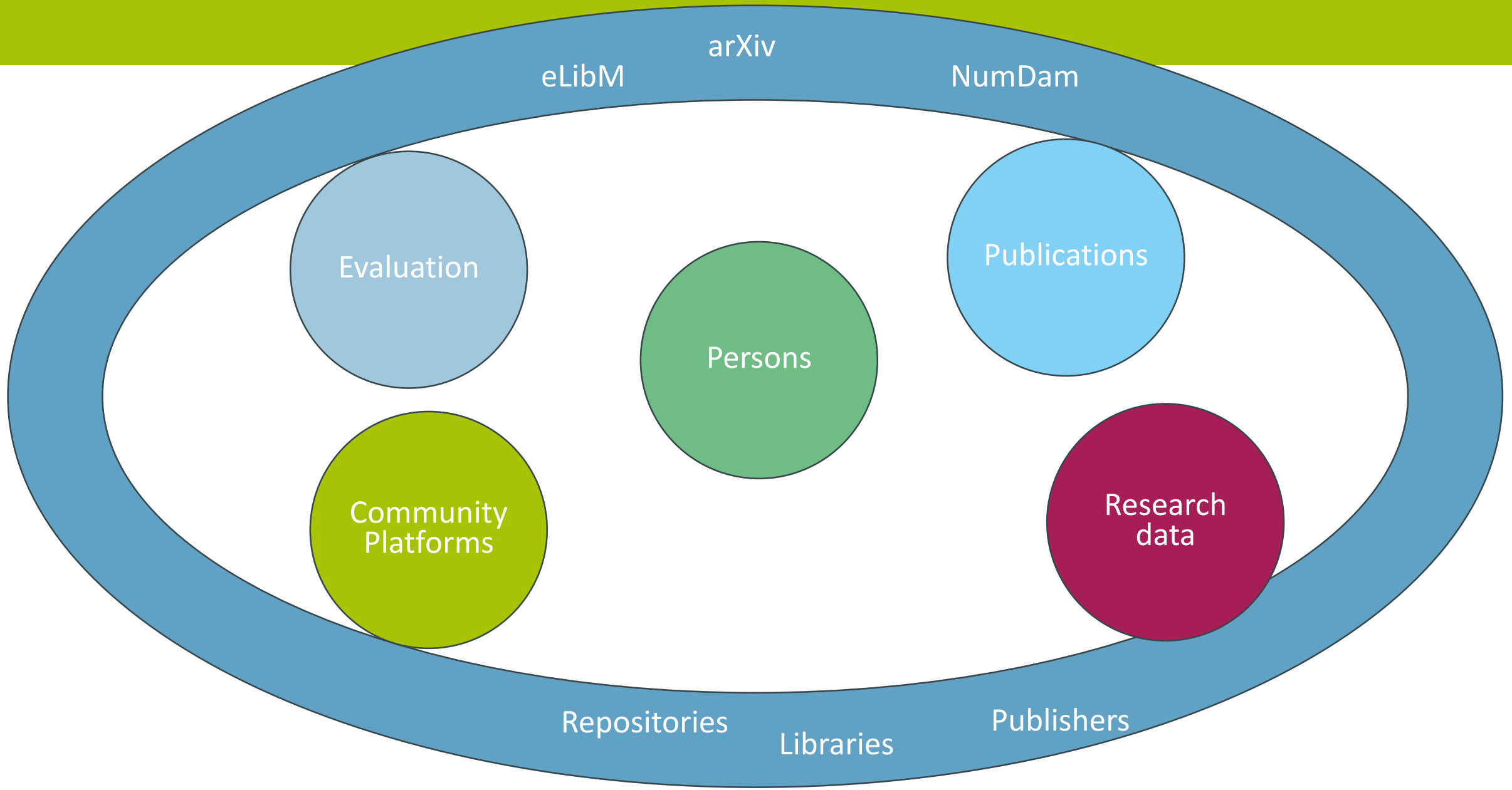
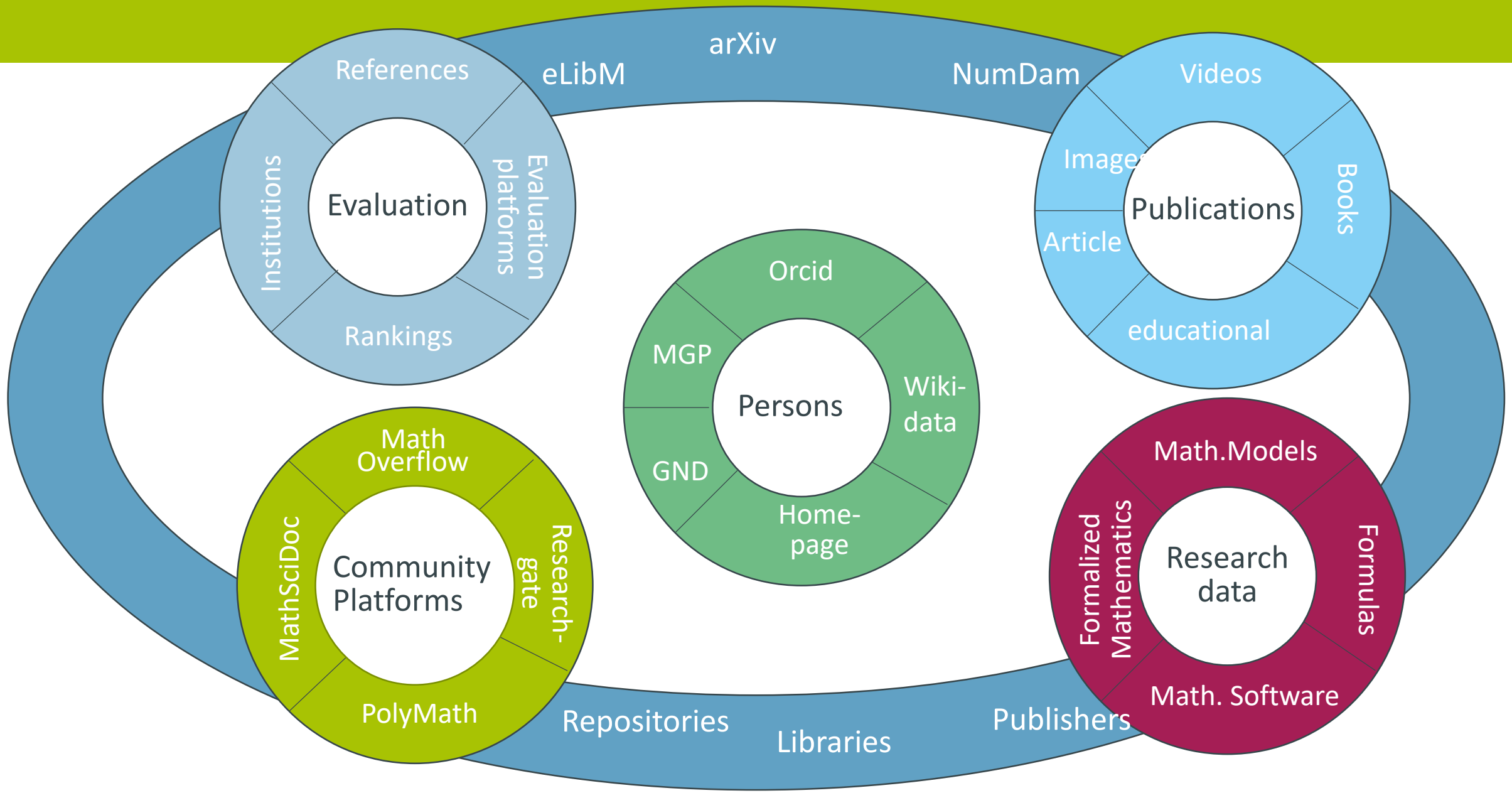
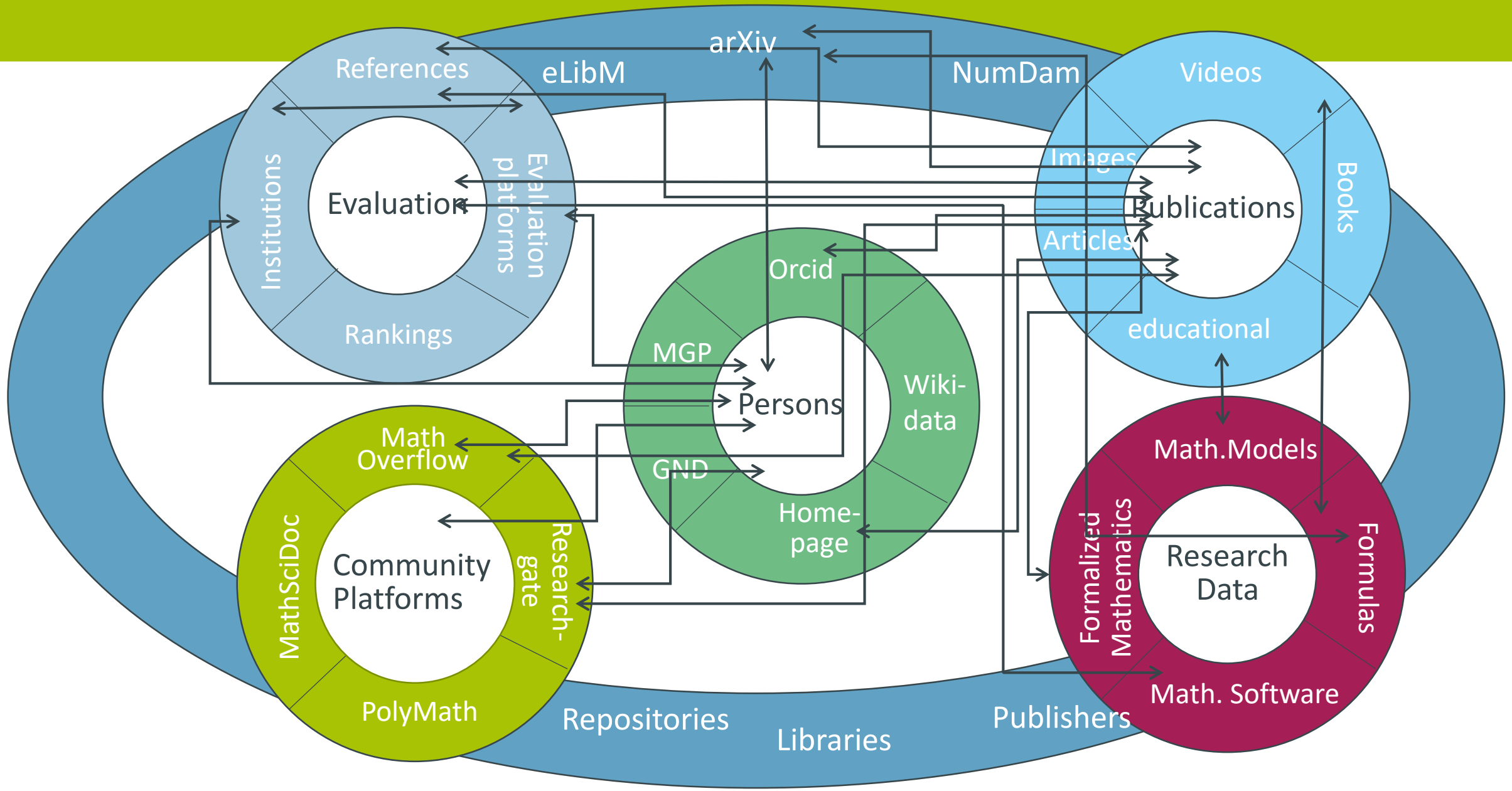
Complex and diverse mathematical information networks



Complex and diverse mathematical information networks



Networked mathematical information networks



What do mathematicians need?

- Easy, comprehensive and open access to multiple sources, including
  - Publications
  - Software
  - Mathematical Research Data
  - Videos and Slides
- Tools to use, evaluate and process these data
  - Support to evaluate the quality of this information
  - Connect diverse information and platforms
  - Tools to identify and analyse relations

What support should we expect from digitization?

- Comprehensive collections (publications, software, data, non-textual material)
- Access and searchability
- Effective interlinking
- Effective support for quality assessment
- Tools to analyse connections
- Support of community activities
- Quick and effective dissemination (including experience from working under COVID-19 restrictions)

Use case 1: Research (Classical literature)

- First information on a mathematical concept (Google, followed by more specific literature research)
- Is the following statement true? (Google to Community platforms such as MathOverflow, links to literature, zbMATH Open, MR)
- In depth literature search (by author, (parts of) title, keyword, journal, combination of different search parameters (zbMATH Open, MR)
- Tools: preprint servers, digital libraries, reviewing platforms (timeline!)
  Ideally: one click access (to the definitive version)
- Aspects: availability of (older) papers, paywalls, version control
- Topic for the future: (efficient) formula search?

Use case 1: Research (Other data)

- Search for mathematical objects (integer sequences, groups, lattices, polytopes, Calabi-Yau manifolds, ... various databases → MaRDI !)
- Search for/use of mathematical software (swMATH, Singular, Macauly, GAP, Pari-GP ... → MaRDI !)

What is important:

- The available information must be **FAIR** (findable, accessible, interoperable, reusable → MaRDI !)
- How do we find/handle research data (most mathematicians need support → MaRDI !) ?

The German government has initiated NFDI. **MaRDI** = Mathemetical Research Data Initiative is the consortium dedicated to mathematical data.

Use case 2: Quality assessment (1)

From the **editor‘s point of view:**

- Is this a case of plagiarism (plagiarism software)?
- Are the results new (referees, zbMATH Open, MR)?
- Are the results correct? (Peer Review (2.0)→ MaRDI, Community platforms)
- How important are the results? (Role of referees/editors)
- Are software and research data handled properly? (→ MaRDI)
- Topic for the future: Can there be tools which help **verify proofs**?

Use case 2: Quality assessment (2)

REFERENCE DOCUMENT CASE 03

As a third result, we have:

Theorem 3 Let $k \geq 1$ be a positive integer Then, there exists a Heron triangle T having $r = k$.

Proof. Let $X = s - a, y = s - b$ and $z = s - c$. Since $r = S/s$ it suffices to show that the equation

$$\frac{xyz}{x+y+z} = k^2$$

has a positive solution x, y, z. We choose $z = 1$ and equation (2) now becomes

$$xy = k^2 (x + y + 1) \quad ,$$

SYNTAX TREE COMPARE SYNTAX TREES 51

or equivalently

$$x (y - k^2) = k^2 (y + 1) \quad ,$$

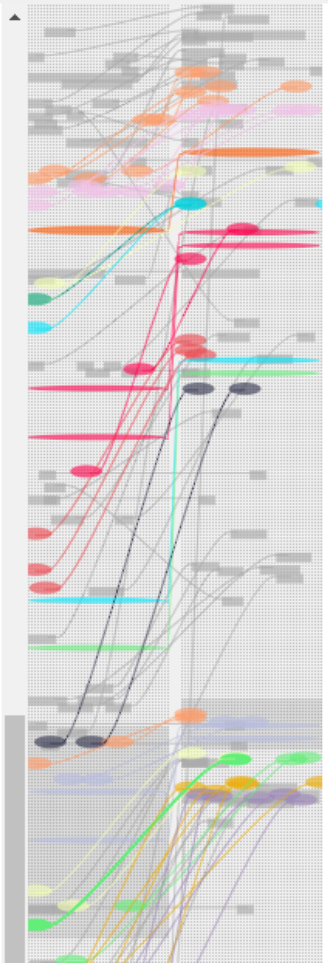
SYNTAX TREE COMPARE SYNTAX TREES 52

or also

$X = \frac{k^2(y+1)}{y-k^2}$. (3)

Clearly, one may choose $y = k^2 + 1$ and then by (3) we have $X = k^4 + 2k^2$. Hence, the triangle of sides

$$a = k^2 + 2, \quad b = k^4 + 2k^2 + 1, \quad c = k^4 + 3k^2 + 1$$



COMPARISON DOCUMENT CASE 03

SIMILAR DOCUMENTS

Proposition 5. Let $k \geq 1$ be a positive integer. Then, there exists a Heron triangle T having $r = k$.

Proof of Proposition 5. We use the notations x, y, z, etc. from the preceeding sections. Since $r = A/s$, it suffices to show that the equation

$$\frac{xyz}{x+y+z} = k^2$$

has a positive solution x, y, z. We choose $z = 1$ and equation (26) becomes

$$xy = k^2 (x + y + 1) \quad ,$$

SYNTAX TREE COMPARE SYNTAX TREES 228

or

$$x (y - k^2) = k^2 (y + 1) \quad ,$$

SYNTAX TREE COMPARE SYNTAX TREES 229

or

$$x = \frac{k^2 (y+1)}{y - k^2}.$$

Clearly, one may now choose $y = k^2 + 1$ and then formula (27) tells us that $x = k^4 + 2k^2$ Hence, the triangle of sides $a = k^2 + 2, b = k^4 + 2k^2 + 1$ and $c = k^4 + 3k^2 + 1$ is a Heron triangle with $r = k$.

While Proposition 5 shows that one can construct Heron triangles of arbitrary integer radius r, this is no longer true if one replaces r by R, but it is almost true. That is, we have the

Use case 2: Quality assessment (3)

From the **researcher‘s point of view:**

Once one has found information (publications, research data, slides, video) the question arises

- How good and reliable is the quality of the information?

This has various aspects:

- Is this the definitive version of a publication? (zbMATH Open, MR)
- Are the results correct? (Role of referee and at this stage also reviewer, involvement of the community, discussion platforms)
- Is this the state of the art? How do I find out about later developments?
- Are the software/data cited and stored properly and reliable? (→MaRDI !)

Use case 2: Quality assessment (4)

zbMATH Open | Documents | Authors | Serials | Classification | Software | Formulæ

Structured Search

an:1465.14002 | Fields | Operators | Help

Mochizuki, Shinichi Recruit

Inter-universal Teichmüller theory. I: Construction of Hodge theaters. (English) Zbl 1465.14002

Publ. Res. Inst. Math. Sci. 57, No. 1-2, 3-207 (2021).

In this series of papers on Inter-Universal Teichmüller Theory [this paper; ibid. 57, No. 1–2, 209–401 (2021; Zbl 1465.14003); Publ. Res. Inst. Math. Sci. 57, No. 1–2, 403–626 (2021; Zbl 1465.14004); ibid. 57, No. 1–2, 627–723 (2021; Zbl 1465.14005)], the author aims to prove the ABC conjecture of Masser and Oesterlé, in close to effective form.

Recall that in its simplest form, the ABC conjecture states that for all $\epsilon > 0$ there is some constant C_ϵ such that for all coprime positive integers a, b, c satisfying $a + b = c$, one has $c \leq C_\epsilon(\prod_{p|abc} p)^{1+\epsilon}$. Here the product runs over all primes p dividing one of a, b and c, but crucially not counting multiplicity. It is arguably the most central open Diophantine problem. For example, applied to $a = x^n$, $b = y^n$ and $c = z^n$ it formally implies that there are at most finitely many counterexamples to Fermat's Last Theorem; if C_ϵ can be made explicit, it thus in principle reduces it to a finite computation. A proof of the ABC conjecture would also lead to a new proof of Mordell's conjecture [*N. D. Elkies*, Int. Math. Res. Not. 1991, No. 7, 99–109 (1991; Zbl 0763.11016)].

A closely related conjecture is the Szpiro conjecture, which relates conductor and discriminant of elliptic curves; the translation is given by passing to the Frey-Hellegouarch curve $y^2 = x(x - a)(x + b)$. The ABC conjecture is known to imply the Szpiro conjecture, but the converse fails, essentially because the discriminant of elliptic curves does not contain an "Archimedean factor". The author observed in [Math. J. Okayama Univ. 52, 1–28 (2010; Zbl 1221.14024)] that if one formulates suitably uniform versions of both conjectures for number fields, they become equivalent. (Additionally, one can arrange for auxiliary congruence conditions at finitely many primes, which is used 2-adically in the present paper.)

With this in mind, the author starts in these papers with the datum of an elliptic curve E over a number field F, satisfying a number of auxiliary conditions; in particular, E is semistable. Moreover, an auxiliary prime ℓ is chosen, of size roughly the square-root of the height of E. Two quantities attached to E at the primes v of bad reduction are of central importance. One is the q-invariant. Recall that by Tate uniformization, $E \times_F F_v$ can be uniformized as $\mathbb{G}_{m,F_v}/q_v^{\mathbb{Z}}$ (as a rigid-analytic variety), for a unique topologically nilpotent unit $q_v \in F_v$. The other invariant are the values of the Θ-function attached to E at certain ℓ-torsion points. These can be described explicitly in terms of q_v, and are basically powers of q_v. The central claim of this series of papers is that through anabelian techniques it is possible to "identify" q-values and Θ-values up to "controlled indeterminacies". This easily gives the desired Diophantine result.

Finally, let me briefly summarize the content of the individual papers. In parts II and III, with the exception of the critical Corollary 3.12, the reader will not find any proof that is longer than a few lines; the typical proof reads "The various assertions of Corollary 2.3 follow immediately from the definitions and the references quoted in the statements of these assertions.", which is in line with the amount of mathematical content. In part I, the first two sections deal with certain group-theoretic results, typical in anabelian geometry, for example about how profinite groups can sit in tempered fundamental groups; these may be of interest to specialists. The rest of part I is largely about the definition of the so-called Hodge-theaters, and some proofs are a bit longer. The category of Hodge theaters has an extremely complicated definition, but the content of these nontrivial proofs is that their category is equivalent to the category with one object and automorphism $\mathbb{Z}/2\mathbb{Z}$, and in fact is canonically equivalent to the category of elliptic curves over F isomorphic to the given E (we note that the functors in both directions are even constructive). In other words, any Hodge theater comes in a unique way from an elliptic curve isomorphic to E. Thus, when the author later chooses an infinite collection of such Hodge theaters, he might as well choose an infinite collection of elliptic curves isomorphic to E. Finally, part IV contains certain technical computations standard in number theory to translate Corollary 3.12 of part III into the ABC conjecture.

Together with *J. Stix*, the reviewer has spent a week in Kyoto to discuss these issues with the author, and has detailed the findings in a manuscript entitled "Why ABC is still a conjecture" [https://www.math.uni-bonn.de/people/scholze/WhyABCisStillaConjecture.pdf] that discusses the issues in slightly more detail. The concerns expressed in this manuscript have not been addressed in the published version.

Reviewer: Peter Scholze (Bonn)

Cited in **3** Reviews
Cited in **1** Document

MSC:

14-02 Research exposition (monographs, survey articles) pertaining to algebraic geometry
14H25 Arithmetic ground fields for curves
14H30 Coverings of curves, fundamental group
14G32 Universal profinite groups (relationship to moduli spaces, projective and moduli towers, Galois theory)

Keywords:

Inter-Universal Teichmüller theory; Hodge theater; global multiplicative subspace; canonical generator; punctured elliptic curve; theta-link; étale theta function; absolute anabelian geometry

MATRIX | AI | CI | PDF | BibTeX | XML | Cite | Full Text: RC | DOI | 1465 r 1 3

References:

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Case 3: Evaluation (1)

This has various **aspects**, such as

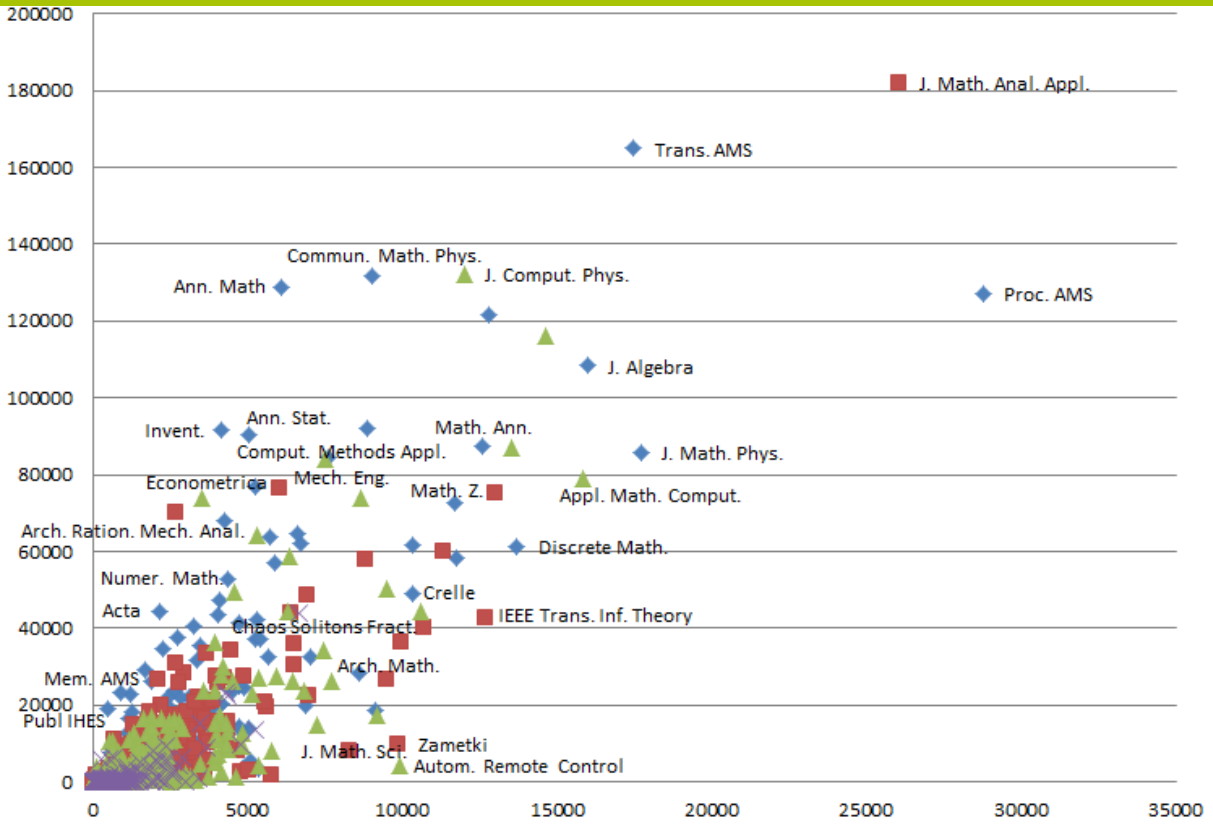
- Evaluation of a submitted article (editorial work, see above)
- Evaluation of a grant proposal
- Promotion committees
- Hiring committees
- Evaluation of departments and institutions

Case 3: Evaluation (2 - Citations and impact)

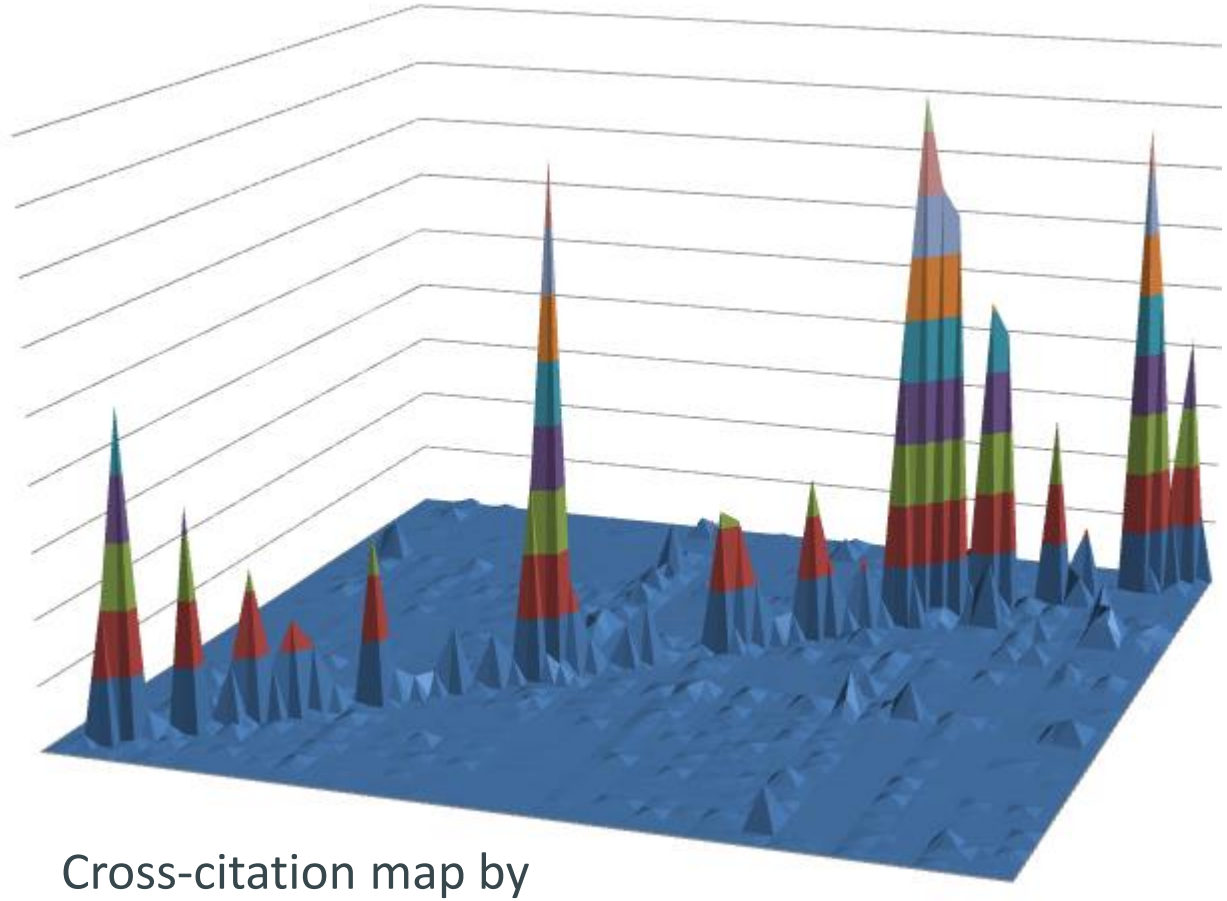
Citations are often used as the gold standard to evaluate the quality of scientific research (of individuals and institutions). This has many aspects:

- Number of citations (in itself a problematic measure)
- Who cites a given work? (Citation circles?)
- Why has an author be cited (part of the historic development, standard work, convenient reference, first appearance of a novel idea, basic tool for new work)?
- Author/institution disambiguation (zbMATH Open, MR)
- Citation context? (ResearchGate)
- Can we have tools wich **help** us evaluate the citation data?

Case 3: Evaluation (3)



Citations and publications figures for different categories of math journals



Cross-citation map by mathematical subjects (MSC)

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Case 3: Evaluation (4 - Research network and knowledge graph)

- **Identify** active research networks (author profiles zbMATH Open: links to orcid, wikidata, Math-Net.Ru, ResearchGate, MPG, GND, homepages, coauthorships)
- Are these open or closed networks (zbMATH Open: collaboration distances)?
- What is the **impact** of a research network outside its own circle?
- Interrelation between the creation of research networks and the evolvement of „hot“ research topics?
- Can we have **tools** to help us analyse research networks and the impact they have?

Case 4 - Dissemination

Electronic tools have revolutionised the dissemination of mathematical knowledge and Covid 19 has accelerated this even further.

- Online teaching (world wide)
- Tools for online/hybrid conferences
- Organising effective discussion fori
- Tools for remote mathematical collaboration (working on a blackboard)
- What will happen to all the videos of seminar and conference talks?

Thank you!

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